Mathematics and Labyrinths.

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Abstract

This is a report on the approaches to labyrinth-like problems from graph theory and knots theory. The problem on bridges of Königsberg and Gauss codes will be considered.

1 Introduction

The aim of my study is to examine the mathematical approaches on problems concerning labyrinths developed in Japan and in China in the 18th and in the 19th centuries.

For this purpose, I will begin with the mathematical development concerning labyrinths in Europe, and then I will research into such a development in Japan and China from a comparative point of view.

Only the very first step will be discussed in this report.

2 Approach by L. EULER

L. EULER [6] solved the problem of 7 bridges of Königsberg, where the bridges were denoted by small letters $a, b, ...$ ("edges" of modern graph theory), the land areas by capital letters $A, B, ...$ ("vertices" as well), and the crossing of a bridge by $AB, ...$ These are considered as the origin of graph theory.

This article by EULER is also considered as one of the origin of topology in which one pays attention not to the positions of bridges and landsbut to the relations of them.

This problem can be related to labyrinths according to my definitions [13] because the path, the start point and the end point are considered.

Although graph theory and topology have the same origin in EULER's work, they are developed to different ways. But in 1970s, some topological approaches were made to graphs. The main subject of them is to embed graphs into topological spaces.

Among them, the approach from knot theory is called spatial graph theory. When one considers a sequence of knots which one crosses in tracing a loop, one can relate the knot to a labyrinth by considering the first knot as the start point of a labyrinth, the last knot as the end point of it and the loop as the path.

Such sequences are called Gauss codes in modern graph theory. The works of C. F. Gauss concerning Gauss codes will be considered in the section 3.

3 Manuscripts of C. F. Gauss

Knot theory is a field of topology. The knots themselves must exist from ancient times, but the mathematical approach to them may have the origin in manuscripts of C. F. Gauss.

Gauss left some manuscripts [7, 8] which led the development in knot theory. I. zur geometria situs [7] consists of 9 parts which are written as a sequence in a notebook, but the time should be different. The parts 1 and 2 may be written between the years 1823 and 1827, and the part 9 may be written after 1840. II. zur Geometrie der Lage für zwei Raumdimensionen [8] may be the foundation for the note dated the 30th of December 1844 written in the page of the part I.2.

In these manuscripts, the upper parts of a loop and the lower one at crossed points are expressed by a sequence of letters, where each letter represents a point, and the order of letters represents the order of passing points when one trace the loop in a direction. Such sequences are called *Gauss codes* later.

P. Rosenstiehl and R. E. Tarjan [19] related a Gauss code to a Hamiltonian cycle.

4 Conclusions

The mathematical approaches to the problems of labyrinths vary according to the problems to be solved. The approaches to the problems of the bridges of Königsberg and of knots were surveyed. Some other approaches by E. Lucas and by P. Rosenstiehl should be considered in future.

Lucas discussed the problems of labyrinth in his books [11, 12]. These works are latter than the manuscripts of Gauss, which were however not appeared in print until the publication of *Werke* in 1900, therefore Lucas wrote them probably without knowing GAUSS's works on knots.

ROSENSTIEHL made approaches to the problems of labyrinths not only from a point of view of graph theory but also from that of topology. Some of his works are related also to spatial graph theory, but the relations between those works and the problems of labyrinths are not yet clear.

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